

Coherent electromagnetic heavy ion reactions: (1) exact treatment of pair production and ionization; (2) mutual Coulomb dissociation

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Some recent theoretical results on coherent electromagnetic processes in ultrarelativistic heavy ion reactions are surveyed.

1. INTRODUCTION: LARGE ELECTROMAGNETIC CROSS SECTIONS

In ultrarelativistic heavy ion collisions, Coulomb induced cross sections are huge, much larger than geometric. For the RHIC case of $100 \text{ GeV} \times 100 \text{ GeV}$ colliding gold ions the predicted cross section for bound-electron positron pairs is about about 110 barns[1]. The corresponding cross section for continuum electron-positron pairs has recently been recalculated to be 34,000 barns[2], consistent with the result of the classic formula of Landau and Lifshitz[3]. The cross section for Coulomb dissociation of the nucleus is about 95 barns[4], and the cross section for ionization of a single electron on one of the ions is about 100,000 barns[5].

2. THE $\delta(z - t)$ POTENTIAL AND ITS SOLUTION

If one works in the appropriate gauge [6], then the Coulomb potential produced by an ultrarelativistic particle (such as a heavy ion) in uniform motion can be expressed in the following form[7]

$$V(\boldsymbol{\rho}, z, t) = -\delta(z - t)\alpha Z_P(1 - \alpha_z) \ln \frac{(\mathbf{b} - \boldsymbol{\rho})^2}{b^2}. \quad (1)$$

\mathbf{b} is the impact parameter, α_z is the Dirac matrix, and the other quantites are the usual coordinates and charge of the moving ion. This the physically relevant ultrarelativistic potential since it was obtained by ignoring terms in $(\mathbf{b} - \boldsymbol{\rho})/\gamma^2$ [7][6].

It was shown in Ref.[8] that the δ function allows the Dirac equation to be solved exactly at the point of interaction, $z = t$. Exact amplitudes then take the same form as perturbation theory amplitudes, but with an effective potential to represent all the higher order effects exactly

$$V(\boldsymbol{\rho}, z, t) = -i\delta(z - t)(1 - \alpha_z)(e^{-i\alpha Z_P \ln(\mathbf{b} - \boldsymbol{\rho})^2} - 1) \quad (2)$$

in place of the potential of Eq.(1).

2.1. Bound electron positron pair production

Early nonperturbative coupled channels calculations at $\gamma = 2.3$ found enhancements of some two orders of magnitude over corresponding perturbation theory calculations of Pb + Pb at small but non-hadronic impact parameters[9]. Improved coupled channels calculations at RHIC energies ($\gamma = 23,000$) indicated approximately a 10% enhancement over perturbation theory results[10]. This result was superseded by calculations employing the exact delta function solution, Eq.(2), resulting in a 2–3% reduction from the perturbation theory result[8]

2.2. Ionization

From impact dependent probabilities for ionization also computed with Eq.(2), cross sections have been calculated[5] for various ion-ion collision combinations in the form $\sigma = A \ln \gamma + B$ where A and B are constants for a given ion-ion pair and γ ($= 1/\sqrt{1-v^2}$) is the relativistic factor one of the ions seen from the rest frame of the other.

The agreement with the Anholt and Becker calculations[11] in the literature is good for the lighter species for both A and B . For heavier ion collisions such as Pb + Pb it is the perturbative energy dependent term A that shows the most discrepancy with Anholt and Becker being about 60% higher. Perhaps this discrepancy is due to the fact that Anholt and Becker use approximate relativistic bound state wave functions and the present calculation utilizes exact Dirac wave functions for the Pb bound states.

CERN SPS data of Pb with a single electron impinging on a Au target has recently been published by Krause et al.[12]. Their measured cross section of 42,000 barns is significantly smaller than the Anholt and Becker calculation (which includes screening) of about 64,000 barns. The result of the present Pb + Pb calculation, which does not include screening, (about 58,000 barns) were privately communicated to Krause et al. and they commented in their paper, “With screening included[11] and scaled to a Au target, the Baltz value agrees with the σ_i measured in the ionization experiment (4.2×10^4 b).”

3. THE TWO CENTER LIGHT CONE CALCULATION OF CONTINUUM PAIRS

Because the singular gauge involves δ functions in $(z-t)$ and $(z+t)$, the amplitude for electron positron pair production can be evaluated in closed form[13]. The technique used builds on the solution of Eq.(2) and involves conventional Green’s functions methods. The amplitude for electron positron pair production can also be evaluated in closed form in the light cone gauge using boundary condition considerations. The amplitudes obtained in both gauges agree.

With a reasonable ansatz for the physical infrared cutoff (i.e. the factors of ω^2/γ^2), the closed form result for the amplitude takes the following form

$$\begin{aligned}
 M(p, q) = & 4\eta^2 \int d^2 k_\perp e^{ibk_\perp} \left(([p_\perp - k_\perp]^2 + \frac{\omega^2}{\gamma^2}) ([k_\perp + q_\perp]^2 + \frac{\omega^2}{\gamma^2}) \right)^{i\eta-1} \\
 & \times \left(\frac{\bar{u}(p, s_f)(1 - \alpha_z)(-\not{k}_\perp + m)v(q, s_i)}{2p^+q^- + k_\perp^2 + m^2} \right. \\
 & \left. + \frac{\bar{u}(p, s_f)(1 + \alpha_z)(-\not{p}_\perp + \not{q}_\perp + \not{k}_\perp + m)v(q, s_i)}{2p^-q^+ + (p_\perp - q_\perp - k_\perp)^2 + m^2} \right). \tag{3}
 \end{aligned}$$

The cutoff comes in response to the spatial region $\rho = \gamma/\omega$ where both the singular and light cone potentials begin to lose their validity. Without the $i\eta$ in the exponent Eq.(3) goes over into the corresponding perturbation theory result of Bottcher and Strayer[14]. In fact, if one squares the exact amplitude Eq.(3) and performs the impact parameter integral before the k_\perp integral, the perturbation theory result is obtained. Thus exact cross sections to specific final states are the same as those of perturbation theory.

On the other hand, if we define an impact parameter dependent probability variable

$$P(b) = \sum_{p,q,spins} \rho |M(p,q)|^2 \quad (4)$$

with ρ the density of states, then $P(b)$ can be understood as the mean number in a Poisson distribution for the number of pairs N created at impact parameter b [15–17]

$$P(N,b) = \frac{[P(b)]^N}{N!} e^{-P(b)} \quad (5)$$

The k_\perp integral must first be performed if $P(b)$ is to be used in Eq.(5), and one finds that multiplicity rates should be reduced from perturbation theory.

In recent parallel work Segev and Wells[18] also note that the exact continuum pair cross section is identical to that of perturbation theory. They go on to note that CERN SPS data of Vane et al.[19] are consistent with perturbation theory: specifically that the cross section for continuum pairs goes as $Z_P^2 Z_T^2$. Correspondingly, Hencken, Trautmann, and Baur[2] calculate multiple pair cross sections reduced from perturbation theory.

4. MUTUAL COULOMB DISSOCIATION

The proposed zero degree calorimeter at RHIC will detect the total neutral energy in very constrained cones downstream of beam crossing. For neutrons of the beam momentum, the detected signal then varies as the number of neutrons.

Detection of neutrons from the correlated forward-backward Coulomb or nuclear dissociation of two colliding nuclei provides a practical luminosity monitor, independent of individual detector set-up, in heavy ion colliders. Single or mutual Coulomb dissociation can be reliably predicted in a Weizsacker-Williams formalism using measured photodissociation data as input. Nuclear dissociation is geometric.

We have calculated[20] the total nuclear plus Coulomb correlated dissociation cross section to be 11.0 barns (in comparison to the 7 barn pure hadronic dissociation cross section) for Au + Au at RHIC. The corresponding cross section for detecting a single neutron both forward and backward at the beam momentum is 0.45 barns. These single neutrons come from Coulomb excitation of the giant dipole resonance and in coincidence provide a clean, well predicted signal useful as a beam luminosity monitor.

Similar slightly larger corresponding cross sections are predicted for Pb + Pb at LHC: 0.53 barns for forward-backward single neutrons and 14.8 barns for total nuclear plus Coulomb correlated dissociation.

Other predicted cross section for RHIC (LHC) have been calculated: dissociation of one of the ions, 102 (227) barns; dissociation of an ion going to one neutron, 49 (106) barns; one neutron in one detector and any neutral in the other 1.35 (1.88) barns. These calculated cross sections could provide complementary roles in luminosity monitoring, especially by exploiting their predicted ratios.

5. SUMMARY AND CONCLUSIONS

Distinctive physical consequences have been shown to arise from the exact ultrarelativistic, semi-classical solution of the Dirac Equation in the $\delta(z - t)$ gauge:

(1) There is no non-perturbative enhancement of bound-electron positron pair production in ultrarelativistic heavy ion collisions.

(2) The beam energy dependent ($\sim \ln \gamma$) part of the ionization cross section decreases with increasing Z of the nucleus ionized.

(3) Exact calculation of continuum electron-positron pair production is equal to the perturbation theory result.

(4) Electron positron multiple pair production is reduced from the perturbation theory result.

Mutual Coulomb dissociation has been suggested as a practical luminosity monitor for RHIC and LHC.

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